Adsorbing Directed Paths in a Wedge

E J Janse van Rensburg

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A q/p-Dyck path is a fully directed path from the origin in the square lattice, confined to the q/p-wedge formed by the Y-axis and the line $Y = (q/p)X$, with last vertex in the line $Y = (q/p)X$, and where (p, q) is a pair of positive integers. If $g_{q/p}(t, z)$ is the generating function of q/p-Dyck paths, and the generating variable for edges (or steps) is t, while z is the generating variable of visits in the line $Y = (q/p)X$, then functional recurrances for $g_{q/p}(t, z)$ are examined, and I show that q/p-Dyck paths adsorb at the critical value of z given by

$$z_{q/p} = \frac{p+q}{gcf(p,q)},$$

where $gcf(p, q)$ is the greatest common factor in p and q.

These results can be extended to a model of directed paths in an r-wedge formed by the Y-axis and the line $Y = rX$, where $r \geq 0$ is a real number. Expressions for the entropic force of the path on the line $Y = rX$ is determined. Finally, I will present briefly some results of adsorbing partially directed paths in a q-wedge.